INTUITIONISTIC FUZZY SETS IN GAMMA-SEMIGROUPS

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Abstract. We consider the intuitionistic fuzzification of the concept of several Γ-ideals in a Γ-semigroup \( S \), and investigate some properties of such Γ-ideals.

1. Introduction

The notion of a fuzzy set in a set was introduced by L. A. Zadeh [10], and since then this concept has been applied to various algebraic structures. K. T. Atanassov [1] defined the notion of an intuitionistic fuzzy set, as a concept more general than a fuzzy set (see also [2]). Using fuzzy ideals, N. Kuroki [5] discussed characterizations of semigroups (see also [6]). K. H. Kim and Y. B. Jun [3] considered the intuitionistic fuzzification of the notion of several ideals in a semigroup, and investigated some properties of such ideals (see also [4]). M. K. Sen and N. K. Saha [9] defined the concept of a Γ-semigroup, and established a relation between regular Γ-semigroup and Γ-group (see also [7], [8]). In this paper, we introduce the notion of an intuitionistic fuzzy Γ-ideal of a Γ-semigroup, and we investigate some properties connected with intuitionistic fuzzy Γ-ideals in a Γ-semigroup.

2. Preliminaries

Let \( S = \{x, y, z, \ldots \} \) and \( \Gamma = \{\alpha, \beta, \gamma, \ldots \} \) be two non-empty sets. Then \( S \) is called a Γ-semigroup if it satisfies

\[
\begin{align*}
&\bullet x\gamma y \in S, \\
&\bullet (x\beta y)\gamma z = x\beta(y\gamma z)
\end{align*}
\]

for all \( x, y, z \in S \) and \( \beta, \gamma \in \Gamma \). A non-empty subset \( U \) of a Γ-semigroup \( S \) is said to be a Γ-subsemigroup of \( S \) if \( UTU \subseteq U \). A left (right) Γ-ideal of a Γ-semigroup \( S \) is a non-empty subset \( U \) of \( S \) such that \( STU \subseteq U \) (\( UTS \subseteq U \)). If \( U \) is both a left and a right Γ-ideal of a Γ-semigroup \( S \), then we say that \( U \) is a Γ-bi-ideal of \( S \).

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a $\Gamma$-ideal of $S$. A $\Gamma$-subsemigroup $U$ of a $\Gamma$-semigroup $S$ is called an interior $\Gamma$-ideal of $S$ if $S\cup U \subseteq U$. A $\Gamma$-bi-ideal of a $\Gamma$-semigroup $S$ is a $\Gamma$-subsemigroup $U$ of $S$ such that $U\cup STU \subseteq U$. Let $L[x]$ denote the principal left $\Gamma$-ideal of a $\Gamma$-semigroup $S$ generated by $x$ in $S$, that is, $L[x] = \{x\} \cup Sx$. A $\Gamma$-semigroup $S$ is said to be regular if, for each $x \in S$, there exist $s \in S$ and $\beta, \gamma \in \Gamma$ such that $x = x\beta s\gamma x$. A $\Gamma$-semigroup $S$ is a left-zero (right-zero) if $x\gamma y = x$ ($x\gamma y = y$) for all $x, y \in S$ and $\gamma \in \Gamma$. A $\Gamma$-semigroup $S$ is said to be left (right) simple if $S$ has no proper left (right) $\Gamma$-ideals. If a $\Gamma$-semigroup $S$ has no proper $\Gamma$-ideals, then we say that $S$ is simple. An element $e$ in a $\Gamma$-semigroup $S$ is called an idempotent if $e\gamma e = e$ for some $\gamma \in \Gamma$. Let $E_S$ denote the set of all idempotents in a $\Gamma$-semigroup $S$.

By a fuzzy set $\mu$ in a non-empty set $X$ we mean a function $\mu : X \rightarrow [0, 1]$ and the complement of $\mu$, denoted by $\bar{\mu}$, is the fuzzy set in $X$ given by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in X$. An intuitionistic fuzzy set (briefly IFS) $A$ in a non-empty set $X$ is an object having the form
$$A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \},$$
where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set $A$, which is a subset of $X$, respectively, and
$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$
for all $x \in X$. An intuitionistic fuzzy set $A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \}$ in $X$ can be identified to an ordered pair $(\mu_A, \nu_A)$ in $I^X \times I^X$ where $I = [0, 1]$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the IFS $A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \}$. Let $\chi_U$ denote the characteristic function of a non-empty subset $U$ of a $\Gamma$-semigroup $S$.

3. Intuitionistic fuzzy $\Gamma$-ideals

In what follows, let $S$ denote a $\Gamma$-semigroup unless otherwise specified.

**Definition 3.1.** For an IFS $A = (\mu_A, \nu_A)$ in $S$, consider the following axioms:

$(\Gamma S_1)$ $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\},$

$(\Gamma S_2)$ $\nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a first (resp. second) intuitionistic fuzzy $\Gamma$-subsemigroup (briefly IFT$S_1$ (resp. IFT$S_2$)) of $S$ if it satisfies $(\Gamma S_1)$ (resp. $(\Gamma S_2)$). Also, $A = (\mu_A, \nu_A)$ is said to be an intuitionistic fuzzy $\Gamma$-subsemigroup (briefly IFT$S$) of $S$ if it is both a first and a second intuitionistic fuzzy $\Gamma$-subsemigroup.

**Theorem 3.2.** If $U$ is a $\Gamma$-subsemigroup of $S$, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an IFT$S$ of $S$. 

Proof. Let \( x, y \in S \) and \( \gamma \in \Gamma \). From the hypothesis, \( x\gamma y \in U \) if \( x, y \in U \). In this case
\[
\chi_U(x\gamma y) = 1 \geq \min\{\chi_U(x), \chi_U(y)\}
\]
and
\[
\bar{\chi}_U(x\gamma y) = 1 - \chi_U(x\gamma y)
\leq 1 - \min\{\chi_U(x), \chi_U(y)\}
= \max\{1 - \chi_U(x), 1 - \chi_U(y)\}
= \max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\}.
\]
If \( x \notin U \) or \( y \notin U \), then \( \chi_U(x) = 0 \) or \( \chi_U(y) = 0 \). Thus
\[
\chi_U(x\gamma y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}
\]
and
\[
\max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\} = \max\{1 - \chi_U(x), 1 - \chi_U(y)\}
= 1 - \min\{\chi_U(x), \chi_U(y)\}
= 1 \geq \bar{\chi}_U(x\gamma y).
\]
This completes the proof. \( \square \)

Theorem 3.3. Let \( U \) be a non-empty subset of \( S \). If \( \bar{U} = (\chi_U, \bar{\chi}_U) \) is an IFTS\(_1\) or IFTS\(_2\) of \( S \), then \( U \) is a \( \Gamma \)-subsemigroup of \( S \).

Proof. Suppose that \( \bar{U} = (\chi_U, \bar{\chi}_U) \) is an IFTS\(_1\) of \( S \) and \( x \in UTU \). In this case, \( x = u\gamma v \) for some \( u, v \in U \) and \( \gamma \in \Gamma \). It follows from (\( \Gamma S_1 \)) that
\[
\chi_U(x) = \chi_U(u\gamma v) \geq \min\{\chi_U(u), \chi_U(v)\} = 1.
\]
Hence \( \chi_U(x) = 1 \), i.e. \( x \in U \). Thus \( U \) is a \( \Gamma \)-subsemigroup of \( S \).

Now, assume that \( \bar{U} = (\chi_U, \bar{\chi}_U) \) is an IFTS\(_2\) of \( S \) and \( x' \in UTU \). Then \( x' = u'\gamma' v' \) for some \( u', v' \in U \) and \( \gamma' \in \Gamma \). Using (\( \Gamma S_2 \)), we get that
\[
\bar{\chi}_U(x') = \bar{\chi}_U(u'\gamma' v')
\leq \max\{\bar{\chi}_U(u'), \bar{\chi}_U(v')\}
= \max\{1 - \chi_U(u'), 1 - \chi_U(v')\} = 0
\]
and so \( \bar{\chi}_U(x') = 1 - \chi_U(x') = 0 \). Therefore \( \chi_U(x') = 1 \), i.e. \( x' \in U \). This completes the proof. \( \square \)

Definition 3.4. For an IFS \( A = (\mu_A, \nu_A) \) in \( S \), consider the following axioms:
(\( \Gamma L\)) \( \mu_A(x\gamma y) \geq \mu_A(y) \),
(\( \Gamma L\)) \( \nu_A(x\gamma y) \leq \nu_A(y) \)
for all \( x, y \in S \) and \( \gamma \in \Gamma \). Then \( A = (\mu_A, \nu_A) \) is called a first (resp. second) intuitionistic fuzzy left \( \Gamma \)-ideal (briefly IFL\(_1\) (resp. IFL\(_2\) \( \Gamma \)ideal) of \( S \) if it satisfies (\( \Gamma L\)) (resp. (\( \Gamma L\))). Also, \( A = (\mu_A, \nu_A) \) is said to be an intuitionistic fuzzy left \( \Gamma \)-ideal (briefly IFL\(_1\)) of \( S \) if it is both a first and a second intuitionistic fuzzy left \( \Gamma \)-ideal.
**Definition 3.5.** For an IFS $A = (\mu_A, \nu_A)$ in $S$, consider the following axioms:

$(RGI_1)$ \[ \mu_A(x\gamma y) \geq \mu_A(x), \]

$(RGI_2)$ \[ \nu_A(x\gamma y) \leq \nu_A(x) \]

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a first (resp. second) intuitionistic fuzzy right $\Gamma$-ideal of $S$ if it satisfies $(RGI_1)$ (resp. $(RGI_2)$). Also, $A = (\mu_A, \nu_A)$ is said to be an intuitionistic fuzzy right $\Gamma$-ideal of $S$ if it is both a first and a second intuitionistic fuzzy right $\Gamma$-ideal.

**Definition 3.6.** Let $A = (\mu_A, \nu_A)$ be an IFS in $S$. Then $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy $\Gamma$-ideal (briefly $IF\Gamma I$) of $S$ if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right $\Gamma$-ideal.

**Proposition 3.7.** Let $U$ be a left-zero $\Gamma$-subsemigroup of $S$. If $A = (\mu_A, \nu_A)$ is an $IF\Gamma I$ of $S$, then the restriction of $A$ to $U$ is constant, that is, $A(x) = A(y)$ for all $x, y \in U$.

**Proof.** Let $x, y \in U$. Since $U$ is left-zero, $x\gamma y = x$ and $y\gamma x = y$ for all $\gamma \in \Gamma$. In this case, from the hypothesis, we have that

\[ \mu_A(x) = \mu_A(x\gamma y) \geq \mu_A(y), \]
\[ \mu_A(y) = \mu_A(y\gamma x) \geq \mu_A(x) \]

and

\[ \nu_A(x) = \nu_A(x\gamma y) \leq \nu_A(y), \]
\[ \nu_A(y) = \nu_A(y\gamma x) \leq \nu_A(x) \]

Thus we obtain $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$ for all $x, y \in U$. Hence $A(x) = A(y)$ for all $x, y \in U$. \hfill \Box

**Lemma 3.8.** If $U$ is a left $\Gamma$-ideal of $S$, then $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I$ of $S$.

**Proof.** Let $x, y \in S$ and $\gamma \in \Gamma$. Since $U$ is a left $\Gamma$-ideal of $S$, $x\gamma y \in U$ if $y \in U$. It follows that

\[ \chi_U(x\gamma y) = 1 = \chi_U(y) \]

and

\[ \tilde{\chi}_U(x\gamma y) = 1 - \chi_U(x\gamma y) = 0 = 1 - \chi_U(y) = \tilde{\chi}_U(y). \]

If $y \notin U$, then $\chi_U(y) = 0$. In this case

\[ \chi_U(x\gamma y) \geq 0 = \chi_U(y) \]

and

\[ \tilde{\chi}_U(y) = 1 - \chi_U(y) = 1 \geq \tilde{\chi}_U(x\gamma y). \]

Consequently, $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I$ of $S$. \hfill \Box

**Theorem 3.9.** Let $A = (\mu_A, \nu_A)$ be an $IF\Gamma I$ of $S$. If $E_S$ is a left-zero $\Gamma$-subsemigroup of $S$, then $A(e) = A(e')$ for all $e, e' \in E_S$.  


Let for an \( \nu \) it is clear that every \( \mu \) if
\[
\mu_A(e) = \mu_A(e' \gamma e) \geq \mu_A(e'),
\]
and
\[
\nu_A(e) = \nu_A(e' \gamma e) \leq \nu_A(e'),
\]
Thus we have \( \mu_A(e) = \mu_A(e') \) and \( \nu_A(e) = \nu_A(e') \) for all \( e, e' \in E_S \). This completes the proof.

**Theorem 3.10.** Let \( S \) be regular. If, for every non-empty subset \( U \) of \( S \), \( \tilde{U} = (\chi_U, \tilde{\chi}_U) \) is an \( IFLI_I \) (or \( IFLT_I \)) of \( S \) and \( \tilde{U}(e) = \tilde{U}(e') \) for all \( e, e' \in E_S \), then \( E_S \) is a left-zero \( \Gamma \)-subsemigroup of \( S \).

**Proof.** Since \( S \) is regular, \( E_S \) is non-empty. Let \( e = e' \gamma e \) where \( \gamma, \gamma' \in \Gamma \). Because of \( S \) is regular, \( L[e] = ST e \). Since \( L[e] \) is a left \( \Gamma \)-ideal of \( S \), we obtain \( \tilde{L}[e] = (\chi_{L[e]}, \tilde{\chi}_{L[e]}) \) is an \( IFLI_I \) (or \( IFLT_I \)) of \( S \) by Lemma 3.8. In this case, from the hypothesis, we get that
\[
\chi_{L[e]}(e') = \chi_{L[e]}(e) = 1 \text{ (or } \tilde{\chi}_{L[e]}(e') = \tilde{\chi}_{L[e]}(e) = 0 \).
\]
Hence \( e' \in L[e] = ST e \). Thus
\[
e' = x \beta e = x \beta (e' \gamma e) = e' \gamma e
\]
for some \( x \in S \) and \( \beta \in \Gamma \). Consequently, \( E_S \) is a left-zero \( \Gamma \)-semigroup.

**Definition 3.11.** For an IFS \( A = (\mu_A, \nu_A) \) in \( S \), consider the following axioms:

- \( (ITI_1) \) \( \mu_A(x \beta s \gamma y) \geq \mu_A(s) \),
- \( (ITI_2) \) \( \nu_A(x \beta s \gamma y) \leq \nu_A(s) \)

for all \( s, x, y \in S \) and \( \beta, \gamma \in \Gamma \). Then \( A = (\mu_A, \nu_A) \) is called a first (resp. second) intuitionistic fuzzy interior \( \Gamma \)-ideal (briefly \( IFTI_I \) (resp. \( IFTI_I \))) of \( S \) if it is an \( IIFTI_I \) (resp. \( IFTS_I \)) satisfying \( (ITI_1) \) (resp. \( (ITI_2) \)). Also, \( A = (\mu_A, \nu_A) \) is said to be an intuitionistic fuzzy interior \( \Gamma \)-ideal (briefly \( IFTI_I \)) of \( S \) if it is both a first and a second intuitionistic fuzzy interior \( \Gamma \)-ideal.

**Remark 3.12.** It is clear that every \( IFTI_I \) of \( S \) is an \( IFTI_I \) of \( S \).

**Theorem 3.13.** If \( S \) is regular, then every \( IFTI_I \) of \( S \) is an \( IFTI_I \) of \( S \).

**Proof.** Let \( A = (\mu_A, \nu_A) \) be an \( IFTI_I \) of \( S \) and \( x, y \in S \). In this case, because of \( S \) is regular, there exist \( s, s' \in S \) and \( \beta, \beta', \gamma, \gamma' \in \Gamma \) such that \( x = x \beta s \gamma x \)
and $y = y' s' \gamma' y$. Thus

$$
\begin{align*}
\mu_A(x \alpha' y) &= \mu_A(x \alpha'(y' s' \gamma' y)) \\
&= \mu_A(x \alpha'y' \beta'(s' \gamma' y)) \\
&\geq \mu_A(y)
\end{align*}
$$

and

$$
\begin{align*}
\nu_A(x \alpha' y) &= \nu_A(x \alpha'(y' s' \gamma' y)) \\
&= \nu_A(x \alpha'y' \beta'(s' \gamma' y)) \\
&\leq \nu_A(y)
\end{align*}
$$

for all $\alpha' \in \Gamma$. It follows that $A = (\mu_A, \nu_A)$ is an $IF\Gamma I$ of $S$. Similarly, we can show that $A = (\mu_A, \nu_A)$ is an $IF\Gamma I$ of $S$. This completes the proof. $\Box$

**Theorem 3.14.** If $U$ is an interior $\Gamma$-ideal of $S$, then $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I$ of $S$.

**Proof.** Since $U$ is a $\Gamma$-subsemigroup of $S$, we have that $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma S$ of $S$ by Theorem 3.2. Let $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. From the hypothesis, $x \beta s \gamma y \in U$ if $s \in U$. In this case

$$
\chi_U(x \beta s \gamma y) = 1 = \chi_U(s)
$$

and

$$
\tilde{\chi}_U(x \beta s \gamma y) = 1 - \chi_U(x \beta s \gamma y) = 0 = 1 - \chi_U(s) = \tilde{\chi}_U(s).
$$

If $s \notin U$, then $\chi_U(s) = 0$. Thus

$$
\chi_U(x \beta s \gamma y) \geq 0 = \chi_U(s)
$$

and

$$
\tilde{\chi}_U(s) = 1 - \chi_U(s) = 1 \geq \tilde{\chi}_U(x \beta s \gamma y).
$$

Consequently, $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I$ of $S$. $\Box$

**Theorem 3.15.** Let $S$ be regular and $U$ a non-empty subset of $S$. If $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I_1$ or $IF\Gamma I_2$ of $S$, then $U$ is an interior $\Gamma$-ideal of $S$.

**Proof.** It is clear that $U$ is a $\Gamma$-subsemigroup of $S$ by Theorem 3.3. Suppose that $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I_1$ of $S$ and $x \in STU S$. In this case, $x = s \beta u \gamma t$ for some $s, t \in S$, $u \in U$ and $\beta, \gamma \in \Gamma$. It follows from $(IF I_1)$ that

$$
\chi_U(x) = \chi_U(s \beta u \gamma t) \geq \chi_U(u) = 1.
$$

Hence $\chi_U(x) = 1$, i.e. $x \in U$. Thus $U$ is an interior $\Gamma$-ideal of $S$.

Now, assume that $\tilde{U} = (\chi_U, \tilde{\chi}_U)$ is an $IF\Gamma I_2$ of $S$ and $x' \in STU S$. Then $x' = s' \beta' u' \gamma' t'$ for some $s', t' \in S$, $u' \in U$ and $\beta', \gamma' \in \Gamma$. Using $(IF I_2)$, we obtain

$$
\tilde{\chi}_U(x') = \tilde{\chi}_U(s' \beta' u' \gamma' t') \leq \tilde{\chi}_U(u') = 1 - \chi_U(u') = 0
$$

and so $\tilde{\chi}_U(x') = 1 - \chi_U(x') = 0$. Therefore $\chi_U(x') = 1$, i.e. $x' \in U$. This completes the proof. $\Box$
Definition 3.16. $S$ is called first (resp. second) intuitionistic fuzzy left simple if every IFL$_1$ (resp. IFL$_2$) of $S$ is constant. Also, $S$ is said to be intuitionistic fuzzy left simple if it is both first and second intuitionistic fuzzy left simple, i.e. every IFL$_1$ of $S$ is constant.

Theorem 3.17. If $S$ is left simple, then $S$ is intuitionistic fuzzy left simple.

Proof. Let $A = (\mu_A, \nu_A)$ be an IFL$_1$ of $S$ and $x, x' \in S$. In this case, because of $S$ is left simple, there exist $s, s' \in S$ and $\gamma, \gamma' \in \Gamma$ such that $x = s\gamma x'$ and $x' = s'\gamma' x$. Thus, since $A = (\mu_A, \nu_A)$ is an IFL$_1$ of $S$, we get that

$$
\mu_A(x) = \mu_A(s\gamma x') \geq \mu_A(x'),
$$

$$
\mu_A(x') = \mu_A(s'\gamma' x) \geq \mu_A(x)
$$

and

$$
\nu_A(x) = \nu_A(s\gamma x') \leq \nu_A(x'),
$$

$$
\nu_A(x') = \nu_A(s'\gamma' x) \leq \nu_A(x).
$$

Hence we have $\mu_A(x) = \mu_A(x')$ and $\nu_A(x) = \nu_A(x')$ for all $x, x' \in S$, that is, $A(x) = A(x')$ for all $x, x' \in S$. Consequently, $S$ is intuitionistic fuzzy left simple. \qed

Theorem 3.18. If $S$ is first or second intuitionistic fuzzy left simple, then $S$ is left simple.

Proof. Let $U$ be a left $\Gamma$-ideal of $S$. Suppose that $S$ is first (or second) intuitionistic fuzzy left simple. Because of $U = (\chi_U, \bar{\chi}_U)$ is an IFL$_1$ of $S$ by Lemma 3.8, $\bar{U} = (\chi_U, \bar{\chi}_U)$ is an IFL$_1$ (and IFL$_2$) of $S$. From the hypothesis, $\chi_U$ (and $\bar{\chi}_U$) is constant. Since $U$ is non-empty, it follows that $\chi_U = 1$ (or $\bar{\chi}_U = 0$), where $1$ and $0$ are fuzzy sets in $S$ defined by $1(x) = 1$ and $0(x) = 0$ for all $x \in S$, respectively. Thus $x \in U$ for all $x \in S$. This completes the proof. \qed

Theorem 3.19. If $S$ is simple, then every IFI$_1$ of $S$ is constant.

Proof. Let $A = (\mu_A, \nu_A)$ be an IFI$_1$ of $S$ and $x, x' \in S$. In this case, because of $S$ is simple, there exist $s, s', t, t' \in S$ and $\beta, \beta', \gamma, \gamma' \in \Gamma$ such that $x = s\beta x'\gamma t$ and $x' = s'\beta' x'\gamma' t'$. Thus, since $A = (\mu_A, \nu_A)$ is an IFI$_1$ of $S$, we obtain that

$$
\mu_A(x) = \mu_A(s\beta x'\gamma t) \geq \mu_A(x'),
$$

$$
\mu_A(x') = \mu_A(s'\beta' x'\gamma' t') \geq \mu_A(x)
$$

and

$$
\nu_A(x) = \nu_A(s\beta x'\gamma t) \leq \nu_A(x'),
$$

$$
\nu_A(x') = \nu_A(s'\beta' x'\gamma' t') \leq \nu_A(x).
$$

Hence we get $\mu_A(x) = \mu_A(x')$ and $\nu_A(x) = \nu_A(x')$ for all $x, x' \in S$. Consequently, $A = (\mu_A, \nu_A)$ is constant. \qed
Definition 3.20. For an $IFTS$ $A = (\mu_A, \nu_A)$ in $S$, consider the following axioms:

$\Gamma B_1$ $\mu_A(x; \beta s \gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\Gamma B_2$ $\nu_A(x; \beta s \gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy $\Gamma$-bi-ideal (briefly $IF\Gamma B$) of $S$ if it satisfies $(\Gamma B_1)$ and $(\Gamma B_2)$.

Remark 3.21. It is clear that every $IF\Gamma I$ of $S$ is an $IF\Gamma B$ of $S$.

Theorem 3.22. If $S$ is left simple, then every $IF\Gamma B$ of $S$ is an $IF\Gamma I$ of $S$.

Proof. Let $A = (\mu_A, \nu_A)$ be an $IF\Gamma B$ of $S$ and $x, y \in S$. In this case, from the hypothesis, there exist $s \in S$ and $\gamma \in \Gamma$ such that $y = s \gamma x$. Thus, because of $A = (\mu_A, \nu_A)$ is an $IF\Gamma B$ of $S$, we have that

$\mu_A(x; \beta y) = \mu_A(x; \beta s \gamma x) \geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$

and

$\nu_A(x; \beta y) = \nu_A(x; \beta s \gamma x) \leq \max\{\nu_A(x), \nu_A(x)\} = \nu_A(x)$

for all $\beta \in \Gamma$. It follows that $A = (\mu_A, \nu_A)$ is an $IF\Gamma I$ of $S$.

Proposition 3.23. If $U$ is a $\Gamma$-bi-ideal of $S$, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma B$ of $S$.

Proof. Since $U$ is a $\Gamma$-subsemigroup of $S$, we obtain that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFTS$ of $S$ by Theorem 3.2. Let $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. From the hypothesis, $x; \beta s \gamma y \in U$ if $x, y \in U$. In this case

$\chi_U(x; \beta s \gamma y) = 1 = \min\{\chi_U(x), \chi_U(y)\}$

and

$\bar{\chi}_U(x; \beta s \gamma y) = 1 - \chi_U(x; \beta s \gamma y) = 0 = \max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\}$.

If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus

$\chi_U(x; \beta s \gamma y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}$

and

$\max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\} = \max\{1 - \chi_U(x), 1 - \chi_U(y)\}$

$= 1 - \min\{\chi_U(x), \chi_U(y)\}$

$= 1 \geq \bar{\chi}_U(x; \beta s \gamma y)$.

Consequently, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma B$ of $S$. 

\hfill \Box
References


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